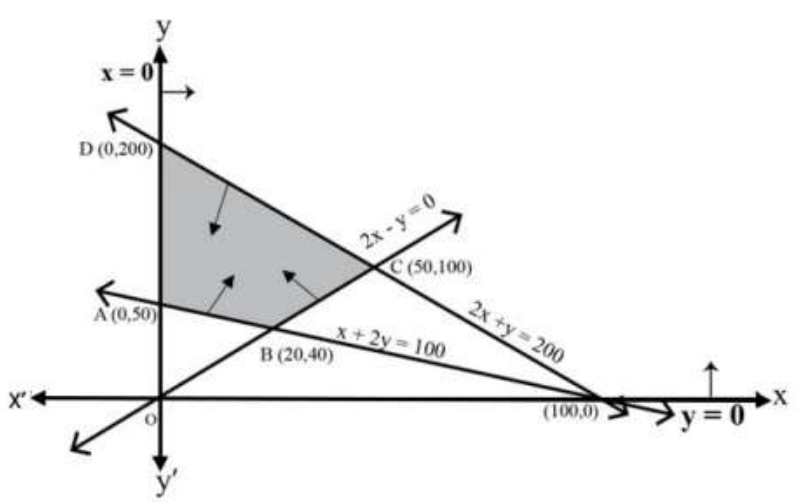
SECOND MODEL EXAMINATION 2023-24

MATHEMATICS (041)- MS

CLASS:XII Max.Marks: 80

MARKING SCHEME			
SET	QN.NO	VALUE POINTS	MARKS SPLIT UP
A	1	b) 1 (a) $x + 2y \leq 5; x + y \leq 4$	1
	2.	(a)	1
	3.	(b)	1
	4.	(d)	1
	5.	(d)	1
	6.	(a) $x + 2y \leq 5; x + y \leq 4$	1
	7.	(a) 2	1
	8.	(d) Z	1
	9.	(b) secx	1
	10.	(b) 1/2	1
	11.	(d)	1
	12.	(b)1	1
	13.	(a) $\pm\frac{2}{15}$	1
	14.	(b) $\frac{\pi}{2}$	1
	15.	(b) 12π	1
	16.	(d) 1	1

	17.	(d) -1	1
	18.	(a) $\frac{1}{2}+\frac{\pi}{4}$	1
	19.	**(d) (A) is false but (R) is true**	1
	20.	**(d) (A) is false but (R) is true**	1
	23 b	$\vec{m} = \vec{a}+\vec{b} = 2\hat{\imath}+\hat{3}\jmath+4\hat{k}$ $\vec{n} = \vec{a}-\vec{b} = -\hat{\jmath}-2\hat{k}$ $\vec{m}$ x $\vec{n} = -2\hat{\imath}+\hat{4}\jmath-2\hat{k}$ $\|\vec{m}$ x $\vec{n}\| = 2\sqrt{6}$ $\vec{l} = \frac{9}{\sqrt{6}}(-\hat{\imath}+\hat{2}\jmath-\hat{k})$	{1/2 ½ ½ ½
	21.b	**OR** π/4 + 3π/4 + π/4 = 5π/4	½ ½ ½ ½ ½ + ½ + ½ + ½
	24.	$\frac{dy}{dx} = e^{x+y}$ $\Rightarrow e^{-y}dy = e^{x}dx$ $\Rightarrow \int e^{-y}dy = \int e^{x}dx$ $\Rightarrow -e^{-y} = e^{x}+C$	1 ½ ½
	26.	$\int_0^4 \|x-1\|\,dx = \int_0^1 (1-x)dx + \int_1^4 (x-1)dx$ 1 $= \left[x-\frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2}-x\right]_1^4$ 1 $= (1-\frac{1}{2}) + (8-4) - (\frac{1}{2}-1)$ $= 5$ 1	½ ½ ½ ½
	22.	**y = a e ^{2x} + b e $^{-x}$** $\frac{dy}{dx}$ = 2**a e ^{2x} - b e $^{-x}$** $\frac{d^2y}{dx^2}$ = 4**a e ^{2x} + b e $^{-x}$**	½ ½

		$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 0$	½ ½
	26.	$f(x) = 4x^3 - 6x^2 - 72x + 30$ implies $f'(x) = 12x^2 - 12x - 72$ $12x^2 - 12x - 72 < 0$ $12(x^2 - x - 6) < 0$ $x^2 - x - 6 < 0$ $x^2 - 3x + 2x - 6 < 0$ $x(x - 3) + 2(x - 3) < 0$ $(x - 3)(x + 2) < 0$ $x \varepsilon (-2, 3)$	3
	28.	$\int \frac{x+3}{(x+5)^3} e^x dx$ $= \int \frac{(x+5)-2}{(x+5^3)} e^x dx$ $= \int \left(\frac{x+5}{(x+5)^3} - \frac{2}{(x+5)^3}\right) e^x dx$ $= \int \left(\frac{1}{(x+5)^2} - \frac{2}{(x+5)^3}\right) e^x dx$ $= \frac{e^x}{(x+5)^2}$	1 ½ 1 ½
	31.	The feasible region determined by the constraints, $x + 2y \geq 100,\ 2x - y \leq 0,\ 2x + y \leq 200,\ x, y \geq 0$, is given below.  $A(0, 50)$, $B(20, 40)$, $C(50, 100)$ and $D(0, 200)$ are the corner points of the feasible region.	1 ½ for 3 lines ½ for shading feasible region ½

The values of Z at these corner points are given below.

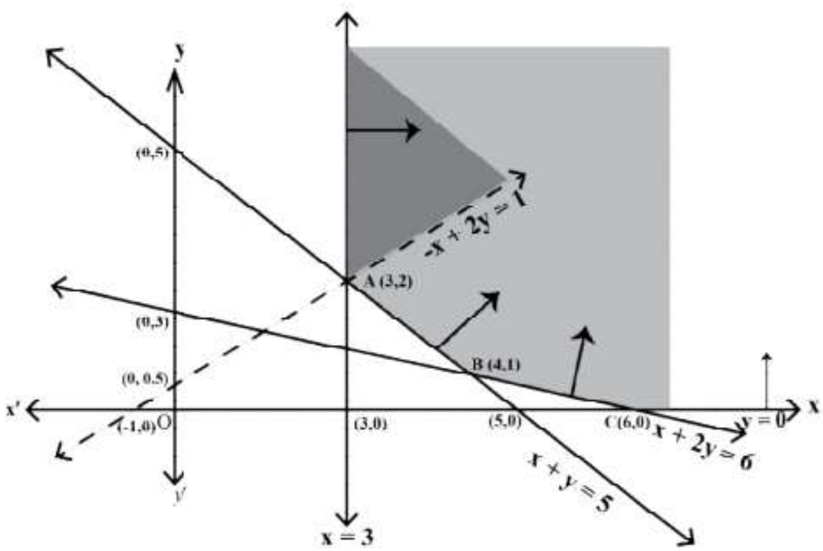
Corner point	Corresponding value of $Z = x + 2y$	
$A\ (0,\ 50)$	**100**	Minimum
$B\ (20,\ 40)$	**100**	Minimum
$C\ (50,\ 100)$	**250**	
$D\ (0,\ 200)$	**400**	

The minimum value of Z is $\mathbf{100}$ at all the points on the line segment joining the points $(\mathbf{0,50})$ and $(\mathbf{20,40})$. 1

OR

The feasible region determined by the constraints, $x \geq 3, x + y \geq 5, x + 2y \geq 6, y \geq 0.$ is given below.

½



1 ½

Here, it can be seen that the feasible region is unbounded.

The values of Z at corner points $A\ (3,\ 2)$, $B\ (4,\ 1)$ and $C\ (6,\ 0)$ are given below.

Corner point	Corresponding value of $Z = -x + 2y$
$A\ (3,\ 2)$	**1** (may or may not be the maximum value)
$B\ (4,\ 1)$	**-2**
$C\ (6,\ 0)$	**-6**

½

Since the feasible region is unbounded, $Z = 1$ may or may not be the maximum value.

Now, we draw the graph of the inequality, $-x + 2y > 1$, and we check whether the resulting open half-plane has any point/s, in common with the feasible region or not. 1 ½

Here, the resulting open half plane has points in common with the feasible region.

Hence, $Z = 1$ is not the maximum value. We conclude, Z has no maximum value.

<table>
<tr><td></td><td></td><td></td><td>½
½ for dotted line
½ for conclusion</td></tr>
<tr><td></td><td>29.</td><td>$ydx + (x - y^2)dy = 0$
we get, $\frac{dx}{dy} + \frac{x}{y} = y$ ½
I.F $= e^{\int P dy} = e^{\int \frac{1}{y} dy} = e^{\log y} = y$ 1
$x.IF = \int Q.IF dy \Rightarrow xy = \int y^2 dy$ 1
$\Rightarrow xy = \frac{y^3}{3} + C$, which is the required general solution ½</td><td>½
½
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½</td></tr>
<tr><td></td><td>30.</td><td>Let X be no. of selected scouts who are well trained in first aid. Here random variable X may have value 0, 1, 2.
Now, $P(X = 0) = \frac{^{20}C_2}{^{50}C_2} = \frac{20 \times 19}{50 \times 49} = \frac{38}{245}$
$P(X = 1) = \frac{^{20}C_1 \times {^{30}C_1}}{^{50}C_2} = \frac{20 \times 30 \times 2}{50 \times 49} = \frac{120}{245}$
$P(X = 2) = \frac{^{30}C_2}{^{50}C_2} = \frac{30 \times 29}{50 \times 49} = \frac{87}{245}$
Now probability distribution table is
<table><tr><td>X</td><td>0</td><td>1</td><td>2</td></tr><tr><td>P(x)</td><td>$\frac{38}{245}$</td><td>$\frac{120}{245}$</td><td>$\frac{87}{245}$</td></tr></table>
OR</td><td>½
½
1
½
½</td></tr>
</table>

		Let A be the event that a student reads Hindi newspaper and B be the event that a student reads English newspaper. P(A) = 60/100 = 0.6, P(B)= 40/100 = 0.4 and P(A ∩ B) = 20/100 = 0.2 (a) Now P(A ∪ B) = P(A) + P(B) – P(A ∩ B) = 0.6 + 0.4 – 0.2 = 0.8 Probability that she reads neither Hindi nor English newspaper = 1 – P(A ∪ B) = 1– 0.8 = 0.2 = 1/5 (b) $P(B/A) = \frac{P(A\cap B)}{P(A)} = \frac{0.2}{0.6} = \frac{1}{3}$ (c) $P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{0.2}{0.4} = \frac{1}{2}$	1 ½ ½ ½ ½
	31. (a) **(b)**	**OR** Let $F(x, y) = \frac{dy}{dx} = \frac{y}{x} - cosec\left(\frac{y}{x}\right)$ $F(\lambda x, \lambda y) = \frac{\lambda y}{\lambda x} - cosec\left(\frac{\lambda y}{\lambda x}\right) = \frac{y}{x} - \text{cosec}\left(\frac{y}{x}\right) = \lambda^{\circ} F(x, y)$ ∴ F(x, y) is a **homogenous function** of degree zero **Putting y = vx** => $\frac{dy}{dx} = x\frac{dv}{dx} + v$ $v + x\frac{dv}{dx} = \frac{vx}{x} - \text{cosec}\left(\frac{vx}{x}\right)$ $\frac{-dv}{cosec\, v} = \frac{dx}{x}$ $\int \frac{-dv}{cosec\, v} = \int \frac{dx}{x}$ $\int -\sin v\ dv = \log\lvert x\rvert + c$ $\cos\frac{y}{x} = \log\lvert x\rvert + C$ **Putting x = 1 & y = 0** => **C = 1** $\cos\frac{y}{2} = \log\lvert x\rvert + 1$ $\cos\frac{y}{x} = \log\lvert ex\rvert$	½ ½ ½ ½ ½ ½ ½

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<tr><td></td><td></td><td></td><td>½

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<tr><td></td><td>34.</td><td>
<table>
<tr><td>x_i</td><td>0</td><td>1</td><td>2</td><td>3</td><td>4</td><td>5</td></tr>
<tr><td>$P(X=x_i)$</td><td>0.2</td><td>k</td><td>2k</td><td>2k</td><td>0</td><td>0</td></tr>
</table>
(i) Since $\sum P = 1 \Rightarrow 0.2 + k + 2k + 2k = 1 \Rightarrow 0.2 + 5k = 1 \Rightarrow 5k = - - 0.2$
$\Rightarrow k = \frac{4}{25}$ 1½ Mark
(ii) $P(X=2) = 2k = \frac{8}{25}$ 1 Mark
(iii) $P(X \geq 2) = 4k = \frac{16}{25}$ 1½ Mark
OR
$P(X \leq 2) = 0.2+3k = \frac{17}{25}$
</td><td></td></tr>
<tr><td></td><td>32.</td><td>
(i) $(x-8)(y+10) = xy \Rightarrow 5x - 4y = 40$
and $(x+16)(y-10) = xy \Rightarrow 5x - 8y = -80$.

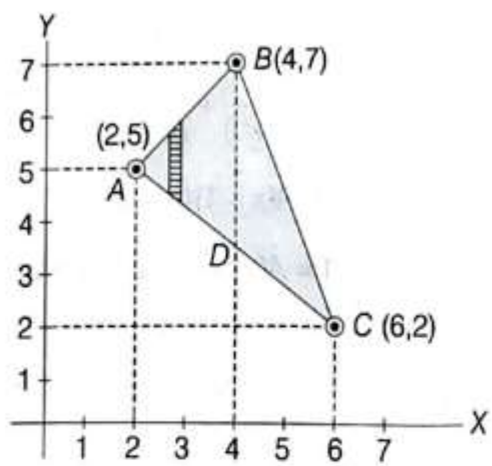
(ii) $\begin{pmatrix} 5 & -4 \\ 5 & -8 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 40 \\ -80 \end{pmatrix}$.

(iii) Let $A = \begin{pmatrix} 5 & -4 \\ 5 & -8 \end{pmatrix}, B = \begin{pmatrix} 40 \\ -80 \end{pmatrix}, X = \begin{pmatrix} x \\ y \end{pmatrix}$
$\because AX = B \Rightarrow X = A^{-1}B$
</td><td>½
½

1

½

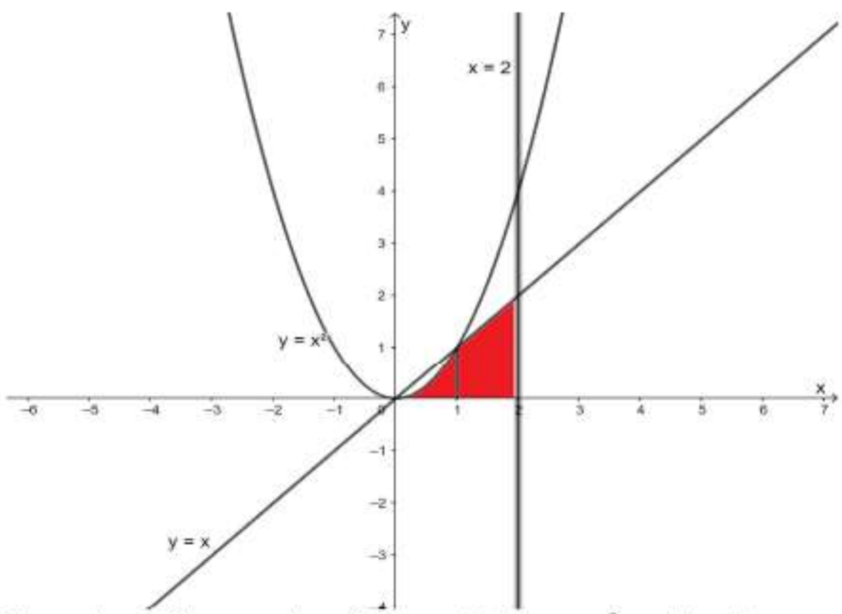
½</td></tr>
</table>

		Now $A^{-1} = \frac{1}{-40+20}\begin{pmatrix} -8 & 4 \\ -5 & 5 \end{pmatrix} = \frac{1}{20}\begin{pmatrix} 8 & -4 \\ 5 & -5 \end{pmatrix}$ $\therefore X = \frac{1}{20}\begin{pmatrix} 8 & -4 \\ 5 & -5 \end{pmatrix}\begin{pmatrix} 40 \\ -80 \end{pmatrix}$ $\Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 32 \\ 30 \end{pmatrix}$ Clearly $x = 32,\ y = 30$. Hence the number of children $= 32$. **OR** As the number of children is 32 and each child gets ₹30. So, total amount distributed by Seema $=$ ₹$(32 \times 30) =$ ₹960.	 ½ ½ ½
	33.	a)dy/dx = 4-x b) x = 4 c) 8	1 1 1 + 1 1 1
	37	 Equations of AB, BC and AC Integration Area = 7 sq. units **OR**	½ 2 2 ½

			2 1 1 1
	35.	$AB = \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$ x – y = 3, 2x + 3y + 4z = 17, y + 2z = 7 $\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$ => A X = C => $X = A^{-1} C$ Since AB = 6I => $A^{-1} = B/6$ $= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$ So X $= \frac{1}{6}\begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} = \begin{bmatrix} 1/2 \\ 17/6 \\ 7/6 \end{bmatrix}$ Thus x = ½, y = 17/6 z = 7/6 **OR** $\|A\| = -1 \neq 0$ so A^{-1} exist For finding correct cofactors Correct $A^{-1} = \begin{bmatrix} 0 & -2 & -1 \\ 1 & 9 & 5 \\ -2 & -23 & -13 \end{bmatrix}$ X= $A^{-1}B$, x= 1, y = 2 , z = 3	1 1 ½ ½ 1 ½ 1 1 ½ 1 ½ 1

	36.	reflexive symmetric Transitive Equivalence	1.5 1.5 1.5 ½
	38.	The given line is $\vec{r} = -\hat{i} + 3\hat{j} + \hat{k} + \lambda(2\hat{i} + 3\hat{j} - \hat{k})$ Its cartesian eq. is $\frac{x+1}{2} = \frac{y-3}{3} = \frac{z-1}{-1} = \lambda\text{(say)}$ ...(i) Any point Q on (i) is $(2\lambda - 1, 3\lambda + 3, -\lambda + 1)$ Also, the given point is $P(5, 4, 2)$. Now d.r's of the line PQ are $(2\lambda - 1 - 5, 3\lambda + 3 - 4, -\lambda + 1 - 2) = (2\lambda - 6, 3\lambda - 1, -\lambda - 1)$. For PQ to be $\perp$ to (i), we must have $(2\lambda - 6)\cdot 2 + (3\lambda - 1)\cdot 3 + (-\lambda - 1)\cdot(-1) = 0$ $\Rightarrow$ $14\lambda - 14 = 0 \Rightarrow \lambda = 1$ $\therefore$ Q is $(1, 6, 0)$ which is the foot of $\perp$ from P on line (i). Now, $PQ = \sqrt{(5-1)^2 + (4-6)^2 + (2-0)^2}$ $= \sqrt{24} = 2\sqrt{6}$ units. Further if $R(\alpha, \beta, \gamma)$ is the image of P in line (i), then $\frac{\alpha+5}{2} = 1, \frac{\beta+4}{2} = 6, \frac{\gamma+2}{2} = 0$ $\Rightarrow$ $\alpha = -3, \beta = 8, \gamma = -2$ $\therefore$ Image of P in line (i) is $R(-3, 8, -2)$.	½ 1 ½ ½ ½ 1 ½ ½

B	**23**	(a) Given $\vec{a}.\vec{b} = \vec{a}.\vec{c}$ $\Rightarrow \vec{a}.\vec{b} - \vec{a}.\vec{c} = 0$ $\Rightarrow \vec{a}.(\vec{b} - \vec{c}) = 0$ $\Rightarrow$ either $\vec{b} = \vec{c}$ or $\vec{a} \perp \vec{b} - \vec{c}$ Also given $\vec{a} \times \vec{b} = \vec{a} \times \vec{c}$ $\Rightarrow \vec{a} \times \vec{b} - \vec{a} \times \vec{c} = 0 \quad \Rightarrow \quad \vec{a} \times (\vec{b} - \vec{c}) = 0$ $\Rightarrow \vec{a} \| \vec{b} - \vec{c}$ or $\vec{b} = \vec{c}$ But vector a cannot be both parallel and perpendicular to (vector b-c). Hence vector b=c.	
B	**25**	$f(x) = 4x^3 - 6x^2 - 72x + 30$ implies $f^{/}(x) = 12x^2 - 12x - 72$ $12x^2 - 12x - 72 < 0$ $12(x^2 - x - 6) < 0$ $x^2 - x - 6 < 0$ $x^2 - 3x + 2x - 6 < 0$ $x(x - 3) + 2(x - 3) < 0$ $(x - 3)(x + 2) < 0$ $x \varepsilon (-2, 3)$	
	26	$\int_0^4 \lvert x-1 \rvert dx = \int_0^1 (1-x)dx + \int_1^4 (x-1)dx$ 1 $= \left[x - \frac{x^2}{2}\right]_0^1 + \left[\frac{x^2}{2} - x\right]_1^4$ 1 $= (1 - \frac{1}{2}) + (8 - 4) - (\frac{1}{2} - 1)$ $= 5$ 1	
B	**29 (a)**	$ydx + (x - y^2)dy = 0$ we get, $\frac{dx}{dy} + \frac{x}{y} = y$ ½ $I.F = e^{\int P dy} = e^{\int \frac{1}{y} dy} = e^{\log y} = y$ 1 $x.IF = \int Q.IF dy \Rightarrow xy = \int y^2 dy$ 1 $\Rightarrow xy = \frac{y^3}{3} + C$, which is the required general solution ½	

B	**36.**	Let $(a,b) \in N \times N$. Then we have ab = ba (by commutative property of multiplication of natural numbers) $\Rightarrow (a,b)R(a,b)$ Hence, R is reflexive. Let $(a,b),(c,d) \in N \times N$ such that (a, b) R (c, d). Then ad = bc $\Rightarrow cb = da$ (by commutative property of multiplication of natural numbers $\Rightarrow (c,d)R(a,b)$ Hence, R is symmetric. Let $(a,b),(c,d),(e,f) \in N \times N$ such that (a, b) R (c, d) and (c, d) R (e, f). Then ad = bc, cf = de $\Rightarrow adcf = bcde$ $\Rightarrow af = be$ $\Rightarrow (a,b)R(e,f)$ Hence, R is transitive. Since, R is reflexive, symmetric and transitive, R is an equivalence relation on $N \times N$.	1 1+1/2 2 ½	
	37(b)	 The points of intersection of the parabola $y = x^2$ and the line $y = x$ are (0, 0) and (1, 1). Required Area = $\int_0^1 y_{parabola}\,dx + \int_1^2 y_{line}\,dx$ Required Area = $\int_0^1 x^2\,dx + \int_1^2 x\,dx$ $= \left[\frac{x^3}{3}\right]_0^1 + \left[\frac{x^2}{2}\right]_1^2 = \frac{1}{3} + \frac{3}{2} = \frac{11}{6}$	(Correct Fig: 1 Mark) ½ 2 1+1/2	

C	**30 (b)**	$xdy - ydx = \sqrt{x^2 + y^2}\, dx$ It is a **Homogeneous Equation as** $\frac{dy}{dx} = \frac{\sqrt{x^2 + y^2} + y}{x} = \sqrt{1 + (\frac{y}{x})^2} + \frac{y}{x} = f\left(\frac{y}{x}\right).$ Put $y = vx$ $\frac{dy}{dx} = v + x\frac{dv}{dx}$ $v + x\frac{dv}{dx} = \sqrt{1 + v^2} + v$ Separating variables, we get $\frac{dv}{\sqrt{1 + v^2}} = \frac{dx}{x}$ Integrating, we get $log\|v + \sqrt{1 + v^2}\| = log\|x\| + logK, K > 0$ $log\left\|y + \sqrt{x^2 + y^2}\right\| = logx^2K$ $\Rightarrow y + \sqrt{x^2 + y^2} = \pm Kx^2$ $\Rightarrow y + \sqrt{x^2 + y^2} = Cx^2$, which is the required general solution	½ ½ ½ 1+1/2	
C				